The Untapped Math Skills of Working Children in India: Evidence, Possible Explanations, and Implications[†]

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Abstract: It has been widely documented that many children in India lack basic arithmetic skills, as measured by their capacity to solve subtraction and division problems. We surveyed children working in informal markets in Kolkata, West Bengal, and confirmed that most were unable to solve arithmetic problems as typically presented in school. However, we also found that they were able to perform similar operations when framed as market transactions. This discrepancy was not explained by children's ability to memorize prices and quantities in market transactions, assistance from others at their shops, reliance on calculation aids, or reading and writing skills. In fact, many children could solve hypothetical transactions of goods that they did not sell. Our results suggest that these children have arithmetic skills that are untapped by the school system.

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It has been widely documented that many children in India lack basic arithmetic skills. According to the Annual Status of Education Report (ASER), a representative survey of primary school age children in rural India, only one in four fifth graders can solve divisions of a three-digit number by a one-digit number with a remainder and just one in two can solve subtractions of a two-digit number from another with a carry-over (ASER, 2017). Similar results have been found in Sub-Saharan Africa (Uwezo, 2015, 2016a, 2016b). Further, studies in several Indian states have documented that children perform well below their grade-level expected performance when they start school, that the gap between expected and actual performance widens during primary and secondary school, and that there is ample variation in students' preparation within each grade (see, for example, Das & Zajonc, 2010; Bhattacharjea et al., 2011; Muralidharan & Zieleniak, 2014; Muralidharan et al., 2017).

Yet, most assessments of arithmetic skills in India, and the developing world more generally, have relied on written subtraction and division exercises as typically presented in school, which require that children remember and correctly execute school-taught algorithms. This is problematic for at least two reasons. First, focusing exclusively on procedures learned in school could under-estimate children's mathematical skills. Studies in developed and developing countries have found that children often depart from procedures learned at school to solve computation exercises (see, for example, Ginsburg, 1977; Groen & Resnick, 1977; Brown & Burton, 1978; Carpenter et al., 1981). Second, understanding why and how children depart from these procedures and devise their own could potentially inform mathematics instruction (Carpenter et al., 1988).

In this study, we present evidence that many Indian children can solve basic arithmetic problems, even if they cannot recall school-taught algorithms. We focus on a specific population to illustrate our point: children working as shopkeepers in informal markets. These children come from low income families, and as such are likely to perform similarly on traditional subtraction and division problems to the average child surveyed in ASER. Yet, their job requires that they solve arithmetic problems on a daily basis (e.g., to calculate total amounts and change). Thus, they are likely to have acquired a working knowledge of arithmetic.

We surveyed 201 children working in 92 informal markets in Kolkata, West Bengal. Consistent with prior studies, we confirmed that most of these children were unable to solve arithmetic problems as typically presented in school. However, we also found that the same children were able to perform similar operations when framed as market transactions. Importantly, their performance on the market problems was not explained by memorization of combinations or prices and quantities, assistance from others at their shops, reliance on calculation aids, or their ability to perform operations in writing. On the other hand, many of these children could use their arithmetic skills to solve hypothetical transactions of goods that they did not sell. These results suggest that children have arithmetic skills that are untapped by the school system and that could be leveraged to teach them abstract arithmetic.

Our study builds on prior work in psychology and anthropology that has documented that children and adults perform better on arithmetic problems anchored in contexts that are relevant to them than on abstract arithmetic problems (see, for example, Reed & Lave, 1979; Lave et al., 1984; Nunes Carraher et al., 1985, 1987; Nunes et al., 1993), and conversely, that formal training on abstract arithmetic does not necessarily prepare individuals to solve problems in everyday life (see, for example, Gay & Cole, 1976; Saxe, 1982; Schliemann, 1984; Nunes Carraher & Schliemann, 1985; Carraher, 1986). This body of research has been fundamental for generating hypotheses of how individuals acquire and develop arithmetic skills.

Our study differs from prior work in at least three ways. First, it is the largest study of its kind in a developing country. Most previous studies have focused on providing rich descriptions of problem-solving strategies of very small samples of children and adults from a small number of locations.[1] Second, partly because of our larger sample size, we are able to examine potential sources of heterogeneity in arithmetic skills within the group of children who are all market participants. Previous studies have been primarily concerned with comparing the skills of children and adults with different educational backgrounds (e.g., street children and school children).[2] This is important because it offers an opportunity to study the mechanisms underlying the patterns we observe. Finally, we explicitly address the question of generalizability (i.e.,

[1] For example, the previous study that most closely resembles ours, conducted by Nunes Carraher et al. (1985), only included five street children in the city of Recife, in the Brazilian state of Pernambuco. A similar study, conducted by Nunes et al. (1993), included just 16 school children in Recife. The largest study of children in a developing country that we know of was conducted by Nunes Carraher and Schliemann (1985) and it included 50 participants. For a discussion of what is known as the "Piagetian interview method", see Piaget and Szeminska (1941).

[2] One possible exception is Nunes Carraher and Schliemann (1985), who compared the problem-solving strategies and errors of public and private school students in Brazil. As it turns out, they do not find any differences between them.

whether the arithmetic skills acquired by children outside of school are also useful in unfamiliar situations). The bulk of prior work seeks to explain how children acquire these skills and why they are useful in the context for which they were developed (Nunes Carraher et al., 1985; Nunes et al., 1993), but few studies ask whether children can use these skills to tackle other problems.

This paper is structured as follows. Section 1 offers an overview of our methods of data collection. Section 2 presents the results from our study. Section 3 discusses the implications of our findings for research and policy.

Methods

Sampling

The sampling frame for our study included all informal markets where children are employed in the city of Kolkata, in the Indian state of West Bengal. However, we could not select a random sample of markets to be included in our study because there is no comprehensive list of such markets, given that child labor is illegal in India.[3] Therefore, we started from a convenience sample of markets and then visited each location to verify that there were children actually working there. This process yielded 92 retail markets in total.

The universe of potential participants included all children working at these markets but once again we faced the problem that there is no roster of all children. In fact, the number and composition of children changes across seasons, days of the week, and times of the day. Thus, we visited markets when they were busiest (in the mornings and evenings) to maximize the probability of finding children working and drew a convenience sample of children. We only approached children who met three criteria: (a) they had to look like they were under 16 (because we were interested in the arithmetic skills of the population covered by ASER and similar tests); (b) they had to be transacting on their own (because we wanted to maximize our chances of selecting children who used arithmetic in their job);[4] and (c) they had to be selling multiple goods (because we wanted to minimize our chances of selecting children who had memorized

[3] The Child and Adolescent Labor Act of 1986 prohibited the employment of children below the age of 14 in a list of hazardous occupations. In 2016, the act was amended to prohibit child labor in *all* occupations. This amendment allows children to help in family businesses, but only when they are not supposed to be in school.

[4] Enumerators verified that the children were receiving money for transactions and returning change by themselves.

combinations of prices and quantities, which is easier to do with a single good). We followed this process and approached 285 children.

Procedure

Our survey had two main parts: the first part entailed three market transactions (to assess children's arithmetic knowledge as applied to their job) and the second part included a number of exercises that both measured the children's ability to solve arithmetic problems as presented in school, and explored several potential explanations why they may know "market" but not "school" math.

In the first part, each child was approached by a pair of "mystery shoppers" (i.e., enumerators pretending to be regular buyers), who purchased two goods from the child (e.g., 300 grams of beans and 200 grams of peas).[5] When the child calculated the total value of the purchase, one shopper asked the child how he/she had arrived at that value (e.g., saying "I expected it to be less"), prompting the child to explain the calculation. If the calculation was correct, the shoppers paid and left. Otherwise, they asked the child to verify his calculation (e.g., saying "Are you sure? How did you get that? Do it again and see").[6] Then, they offered the child more money than the amount due (so that the child had to calculate the change), took the change, and left. They were followed by two other pairs of mystery shoppers who repeated the same sequence. The plan was for each child's performance to be measured based on three different market transactions.

In the second part, which began after the third transaction, a pair of mystery shoppers approached the child again, revealed that they were conducting a survey, and asked the child whether he or she would be willing to participate. At this stage, enumerators sought informed consent from the child and his/her guardian, if present. If the guardian was not present,

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[5] Shoppers avoided asking "common" amounts (e.g., half a kilogram, a kilogram, or two kilograms) to minimize the possibility of children memorizing combinations of prices and quantities. Whenever a child insisted on the shopper taking a more common amount (e.g., "Why don't you take 500 grams instead of 400 grams?"), the shopper insisted on taking the requested amount. Each pair of shoppers also made sure to ask for different amounts, so that no child would be assessed twice using the same amount.

[6] However, shoppers were instructed not to point out the specific calculation error or hint at the correct answer. If the child answered correctly after he/she was prompted to verify his/her calculations, his/her answer was marked as "correct after mistakes". If he/she answered incorrectly even after the prompt, his/her answer was marked as "incorrect", the shopper indicated what the correct amount would be, and he/she tendered this amount.

enumerators called him/her on their cell phones or returned at a later time when they were around. If consent was granted, enumerators asked the child his/her questions about their background characteristics and arranged a time to conduct the assessments.

The survey took about 10 minutes to complete. After the survey, children were compensated with 200 Rupees (about 3 U.S. dollars). Children were not told about this compensation at the time that they agreed to participate in the survey because we wanted to measure their performance without any additional incentives. The money was given to the guardian (if present) and otherwise to the child.

Instruments

The entire survey, including both parts described above, is included in online Appendix A. In the first part, enumerators recorded the quantities and prices of the goods that they purchased, as well as whether the child calculated the total value of the transaction correctly and returned the correct change. They also recorded whether he/she performed the computations by him/herself, and whether he/she used any calculation aids.

The second part included two brief assessments: (a) a written assessment of number recognition, subtraction, and division; (b) an oral assessment of children's abstract arithmetic skills and of their ability to solve hypothetical market transactions involving goods they did not sell.

For the written assessment, we used one of the publicly-available versions of the Annual Status of Education Report (ASER) math test, developed by *Pratham*, the largest education non-profit in India. We selected this test for two main reasons. First, it produces very clear information on what children know and are able to do. Second, since the ASER test is administered annually to a representative sample of rural households, we can benchmark the results of our study against those of the average (rural) child in West Bengal.

The written assessment was administered as follows (for more details, see ASER, 2014). The enumerator gave the child a sheet with eight single-digit numbers, 10 double-digit numbers, eight subtractions of a two-digit number by another, and four divisions of a three-digit number by a one-digit number. The child was also given pencil in case he/she wanted to use them.[7] Then, the enumerator asked the child to choose two subtraction problems and solve them. If the child

[7] This was also the case in the study conducted by Nunes Carraher et al. (1985) to document children's strategies to solve the arithmetic problems, as well as any errors they may commit.

solved both problems correctly, he/she was asked to choose and solve a division problem. If the child solved it correctly, he/she was categorized as being at “division” level. If the child solved it incorrectly, he/she was categorized as being at “subtraction” level. Conversely, if the child solved one or no subtraction problem correctly, he/she was asked to choose and recognize five two-digit numbers. If the child could recognize at least four numbers, he/she was categorized as being at “two-digit number recognition” level. If the child could not, he/she was asked to choose and recognize five one-digit numbers. If the child could recognize at least four numbers, he/she was categorized as being at “one-digit number recognition” level. Otherwise, he/she was labeled as “beginner”.

The oral assessment contained three sets of questions. The first set included a subtraction and a division exercise very similar to the ones in ASER (e.g., “What do we get if we subtract 29 from 53?”) We included these questions to check whether children performed poorly in the written assessment because they had difficulty reading the exercises or writing their answers or because they had some automatic negative association with written math problems.[8] The second set also included a subtraction and a division exercise, but they were phrased as word problems anchored on objects and/or money (e.g., “Something costs 3 Rupees each. I have 24 Rupees. How many can I get?”)[9] We included these questions to check whether children performed poorly in the written assessment because the numbers were not associated with anything concrete.[10] The final set included two hypothetical market transactions (e.g., “Oil is 100 Rupees a kilogram and rice is 80 Rupees a kilogram. How much should I pay for 300 grams of oil and

[8] To our knowledge, this is the first exercise of its kind in India. In 2006, *Pratham* conducted a study in which children in grades 3 to 8 were asked to read a two-paragraph word problem and solve it (see ASER, 2007). However, children’s performance on that study depended on both, their reading comprehension and arithmetic skills, and we wanted to focus exclusively on the latter.

[9] This exercise resembles one administered by Pratham in 2007, in which children ages 6 to 14 were asked to listen to and solve two subtraction problems involving Indian Rupees (see ASER, 2008).

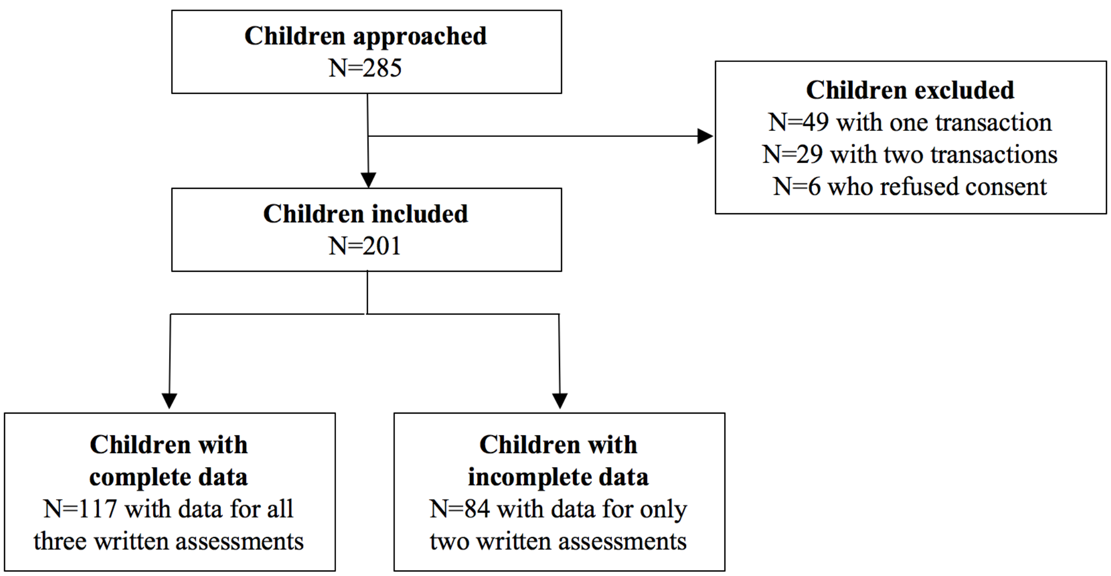
[10] In their study of Brazilian child street vendor, Nunes Carraher et al. (1985) found that children performed differently in operations presented as algebraic problems than in similar operations phrased as a word problem. In their study of Liberian tailors, Reed and Lave (1979) found that operations of coins and bills were not perceived as manipulation of numbers, and that this population performed much better on these operations than on similar arithmetic problems involving abstract numbers.

800 grams of rice?")[11] We included these questions to check whether children could apply their arithmetic skills to market transactions with unfamiliar goods and/or prices.

Attrition

Figure 1 shows the number of children in each stage of our study. As the figure shows, we approached 285 children, but we only include 201 of those in our analysis. We exclude 84 children with incomplete data: 49 left their shop after the first transaction in our survey, 29 did so after the second transaction, and six completed all transactions but declined to participate in the assessments. As the figure also shows, we added the last two questions of the written assessment after the study had started, so we only have 117 children with responses to these questions.

Figure 1. Number of children in the study



Notes: The figure shows the number of children who participated in the study: 285 children were approached, of whom 201 were included in the study. Out of those 201 children, 117 have data for all parts of the survey.

We checked whether the children included in and excluded from the study differed on their performance on the market transactions.[12] Included children were more likely than excluded

[11] In the second question, the prices were an order of magnitude higher to check whether children could keep track of larger numbers.

children to do the calculation correctly in all three rounds of market transactions. They were also more likely to get to the correct answer in the first two rounds after the enumerators told them that their calculation was wrong. Finally, included children were also more likely to be selling by themselves (see Table B.1 in online Appendix B). Thus, the findings from our study are likely to be representative of the better-performing children in the markets we visited.

Analysis

We conduct three types of analyses in this paper. First, we compare children's performance on the market transactions and the written assessment. Then, we consider several potential explanations for why children might perform well on the former, but not on the latter. Finally, we explore whether children can apply their arithmetic skills to hypothetical market transactions.

Results

Background Characteristics

Table 1 describes the 201 children included in our analyses. The vast majority (84%) was male and the average child was 12 years old. Most children had gone through the formal school system. More than two thirds (70%) were enrolled in school and nearly all (98%) had some schooling. In fact, over half of them (53%) had completed primary school and even more (59%) attended private tuition. Yet, children in our study were far more likely to have dropped out of school (28%) than the average child of comparable age in West Bengal: only 2% of children ages 6 to 14 in rural areas have dropped out (ASER, 2017). This suggests that lower-performing students are more likely to be recruited by their families to work in markets.

The three most common items sold by children were vegetables (34%), stationary (16%), and fruits (15%). Nearly half (49%) of children sold items by the unit and the other half by kilograms.

Table 1. Children's background characteristics

[12] We could not check the equivalence of these two groups on children's background characteristics or on the written and oral assessments because these data were collected after the three rounds of market transactions, once children (and/or their guardians) had granted consent.

Variable	*Mean*	*S.D.*
Age	11.935	(1.836)
Female	.159	(.367)
Attends school	.701	(.459)
Dropped out of school	.284	(.452)
Never enrolled in school	.015	(.122)
Completed primary school	.527	(.5)
Attends private tuition	.592	(.493)
Number of months selling	19.871	(14.221)
Sells vegetables	.338	(.474)
Sells stationary	.159	(.367)
Sells fruits	.149	(.357)
Sells groceries	.095	(.293)
Sells clay items	.05	(.218)
Sells cigarettes	.05	(.218)
Sells by unit	.488	(.501)
Sells by kilogram	.512	(.501)
N	201	

Notes: (1) Table shows means, standard deviation in parenthesis, and number of non-missing observations. (2) Primary completion is coded as having completed grade 6. (3) Only most frequently sold items are shown. (4) Goods sold by unit include: cigarettes, clay items, *dashakarma* (supplies for a religious ceremony), detergent, fast food, flower plants, garland, groceries, *khaini* (chewing tobacco), betel leaves, pens, knives, stationary, sweets, tea, and *bhujia* (fritters). Goods sold by kilogram include: coal, fish, fruits, rice, seeds, and vegetables. (5) Only one child sold rice and only one other child sold fish, which is why the hypothetical market transactions that we discuss below were new to nearly all children.

"School" Arithmetic Skills

Consistent with prior studies, we found that most children in our sample were unable to solve arithmetic problems as typically presented in school. Table 2 shows the share of children at each level of the written assessment, as described in the Methods section. As the table indicates, nearly a third of the children were at division level and about a fifth at subtraction level. These figures are similar to those of the average student in grades 5 and 6 (the modal grades for 12-year-olds) in rural areas in West Bengal, which are 29% and 19%, respectively (ASER, 2017).

Table 2. Children's performance on the written assessment

Variable	*Mean*	*S.D.*
Child is at division level	.323	(.469)
Child is at subtraction level	.214	(.411)

Child is at double-digit number recognition level	.308	(.463)
Child is at single-digit number recognition level	.134	(.342)
Child is at beginner level	.02	(.14)
N	201	

Notes: Table shows means, standard deviation in parenthesis, and number of non-missing observations.

We expected children who were enrolled in school at the time of the survey to perform better in the written assessment than those who were out of school, either because they are more frequently exposed to the type of arithmetic problems included in the written assessment or because they have self-selected to stay on school. As Table 3 indicates, this is exactly what we found: children enrolled in school were more likely to be classified at division and subtraction level, and less likely to be classified at double- or single-digit recognition, or at the beginner level. All of these differences were statistically significant.[13] This association remains virtually unchanged when we account for children's age. These results may seem unsurprising, but prior comparisons of children with different schooling experiences in other developing countries found little evidence of differences in their performance in arithmetic problems (see Nunes Carraher et al., 1985; Nunes et al., 1993).

Table 3. Children's performance the written assessment by school enrollment

Variable	*Out of school*	*In school*	*Difference*	
Child is at division level	.033	.446	.413***	.432***
	(.181)	(.498)	(.066)	(.064)
Child is at subtraction level	.083	.269	.186***	.191***
	(.278)	(.445)	(.062)	(.062)
Child is at double-digit number recognition level	.466	.241	-.225***	-.234***
	(.503)	(.429)	(.069)	(.069)
Child is at single-digit number recognition level	.35	.042	-.307***	-.317***
	(.062)	(.202)	(.048)	(.047)
Child is at beginner level	.066	0	-.066***	-.070***
	(.251)	(0)	(.021)	(.02)
Age controls?			N	Y
N	60	141	201	201

[13] Note that this could either be a selection (i.e., children with better arithmetic skills are sent to school) or a treatment effect (i.e., children learn formal arithmetic skills because they attend school).

Notes: (1) Columns 1 and 2 show means, standard deviation in parenthesis, and number of non-missing observations. Column 3 shows the difference between both groups from an ordinary least squares regression of the outcome variable on the dummy variable for ongoing school enrollment and its associated standard error. Column 4 shows the same difference, controlling for children's age at the time of the survey. (2) * p<.1, ** p<.05 *** p<.01.

"Market" Arithmetic Skills

Unlike prior studies in India, which have focused on children's (lack of) proficiency in school-taught algorithms, we also found that *nearly all* children in our sample were able to solve arithmetic problems in the context of market transactions, even when these problems were at a similar level of difficulty to those presented in the written assessments. Table 4 shows the share of children who calculated the total amount due correctly, correctly after mistakes, and incorrectly on the three market transactions described in the Methods section. As the table indicates, 88% of children calculated the total amount due correctly in transaction 1, 92% did so in transaction 2, and 91% did so in transaction 3. In fact, if we group together children who calculated the amount due correctly and correctly after mistakes, nearly all children were capable of multiplying the price of each good by the requested quantity of a good, doing it again for a separate good, and adding the result from both multiplications.[14]

Table 4. Children's performance on market transactions

	Transactions		
	1	*2*	*3*
Child calculated amount due correctly	.876	.915	.905
	(.331)	(.279)	(.293)
Child calculated amount due correctly after mistakes	.075	.05	.07
	(.263)	(.218)	(.255)
Child calculated amount due incorrectly	0	.01	.005
	(0)	(.1)	(.071)
Child was helped by someone else	.05	.025	.02
	(.218)	(.156)	(.14)
N	201	201	201

Notes: (1) Table shows means, standard deviation in parenthesis, and number of non-missing observations. (2) "Child responded correctly" category refers to children who handled the transaction correctly without making any mistakes. "Child responded correctly after mistakes" category refers to children who handled the

[14] Unfortunately, enumerators did not record the share of children who calculated the change correctly.

transaction correctly after the mystery shopper asked them to revise their calculations. (3) All rows in this table are mutually exclusive categories.

We need to be cautious in comparing the results from the market transactions and written assessment results since we did not use the same operations or numbers in both cases. Thus, the discrepancy in children's performance may be partly due to differences in the difficulty of the questions. However, we believe this is unlikely to explain our results because the two sets of problems are quite similar in their difficulty level. The written assessment included two two-digit subtractions with carry-over and a three-digit-by-one-digit division with remainder. The transactions with goods sold by kilos involved multiplication and division (e.g., to determine the cost of a fraction of a kilo). Those with goods sold by units only involved multiplication. In both cases, the sub-totals from the two goods purchased had to be added up and the total, which was typically not a multiple of 10 Rupees, had to be subtracted from the amount paid, which was always a multiple of 10 Rupees. Formally, this is a two- (or more) digit subtraction problem with carry-over. Given that the amounts of money tendered and the total cost of the purchases varied across the transactions, these are all distinct subtraction problems.[15]

Potential Explanations for Differences between "School" and "Market" Arithmetic Skills

We designed our survey to rule out several potential explanations for the discrepancy between children's performance on the written assessment and the market transactions. First, we conducted three different transactions with differing quantities demanded (instead of just one) to minimize the possibility that children were relying on memory (rather than calculating) the combinations of goods, prices, and quantities purchased.[16] As Table 4 shows, children's performance was similar across all three transactions, and most answered all of them correctly.[17]

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[15] Further, several studies have found that the addition and multiplication skills of Indian children are highly correlated with their subtraction and division skills, respectively. In fact, it was these studies that led Pratham, the non-profit that designed the written assessment we used, to focus on subtraction and division (ASER, 2017, p. 292).

[16] As we discuss in the Procedure section, the amounts were not common (e.g., 300 grams instead of 500 grams) to minimize the chance that children had memorize combinations of prices and quantities.

[17] Interestingly, the share of children correctly calculating the amount due on the first transaction was slightly lower (88%) than on the second (92%) and third (91%) transactions. We cannot determine why this was the case. One possibility is that our enumerators did not ask for frequently-requested quantities of the goods they purchased (e.g., instead of buying one kilogram, they bought half a kilogram, or a quarter of a kilogram), which might have thrown

Second, we asked enumerators to record whether children received help from others at their shops when calculating the amount due in the transactions. As Table 4 shows, this only occurred in 2% to 5% of the cases. In fact, 63% of children in our study were alone during the transactions. Therefore, their performance is not explained by receiving help from others.

Third, we also asked enumerators to record whether children used any calculation aids (specifically, paper and pencil) when computing the amount due in each transaction. This was not the case for *any* children in *any* of the three transactions. In this respect, the children in our study differ from the adult tailors assessed by Reed and Lave (1979) who solved arithmetic problems primarily by relying on counters and other calculation aids, and resembled more closely the child street vendors in Brazil assessed by Nunes et al. (1993), who handled most transactions without paper and pencil.

Fourth, as mentioned above, we also included an abstract subtraction and division question in the oral assessment that was similar to those in the written assessment to check whether children react differently to written and oral questions.[18] As Table 5 shows, this is not what we found. Only 43% of children answered the oral subtraction question correctly, either on their first try or after making a mistake. This percentage is lower than the 54% of children who answered the subtraction question correctly in the written assessment (see Table 2 above). Further, only 3.4% of children answered the oral division question correctly. This percentage is much lower than the 32% of children who answered the division question correctly in the written assessment. Thus, we found no evidence that children's ability to read and write or their psychological reaction to the kinds of written problems they encountered in school adversely influenced their performance on the written assessment.

Table 5. Children's performance on oral subtraction and division

	Subtraction	*Division*
Child responded correctly	.308	.034
	(.464)	(.182)
Child responded correctly after mistakes	.12	0

off the children the first time around. Another possibility is that, because our enumerators were asking for fractions of the goods being purchased, the first transaction had a "treatment" effect that activated children's arithmetic skills.

[18] As we discuss in the Methods section, we added this question after the survey had started, so we only have responses from 117 of the 201 children included in the study.

	(.326)	(0)
Child responded incorrectly	.393	.359
	(.491)	(.482)
Child was helped by someone else	0	0
	(0)	(0)
Child did not try to answer	.179	.607
	(.385)	(.491)
N	117	117

Notes: (1) Table shows means, standard deviation in parenthesis, and number of non-missing observations. (2) Subtraction question: "What do we get if we subtract 29 from 53?". Division question: "What do we get if we divide 413 by 3?"

Finally, we included a subtraction and division problem in the oral assessment that were anchored on objects and/or money to check whether the abstract nature of the algorithmic manipulation typically taught at school was affecting children's performance. As Table 6 shows, we found that children's performance on these problems was more similar to that on the transactions than on the written assessment. However, it should be noted that the division question in the oral assessment was easier than those in the written assessment (specifically, it was a division of a two-digit number by a one-digit number with no remainder).

This result provides suggestive evidence in favor of the argument made by Reed and Lave (1979) that the algorithmic manipulations taught at school are "divorced from reality" (p. 442). It is also consistent with their finding that Liberian tailors approached arithmetic problems with money differently than similar formal arithmetic problems.

Table 6. Children's performance on subtraction and division of objects and/or money

	Subtraction	*Division*
Child responded correctly	.761	.761
	(.429)	(.429)
Child responded correctly after mistakes	.137	.085
	(.345)	(.281)
Child responded incorrectly	.085	.103
	(.281)	(.305)
Child was helped by someone else	0	0
	(0)	(0)
Child did not try to answer	.017	.051
	(.13)	(.222)
N	117	117

Notes: (1) Table shows means, standard deviation in parenthesis, and number of non-missing observations. (2) Subtraction question: "Assume you have 25 items. I bought 17 items. How many remain?" Division question: "Assume I have 24 Rupees. I want to buy some items at 3 Rupees per unit. How many can I buy?"

Generalizability of "Market" Arithmetic Skills

Having verified that children's performance in the market transactions was not explained by factors unrelated to their arithmetic skills, we wanted to understand whether they could apply these skills to unfamiliar situations. Therefore, we presented them with two hypothetical market transactions; one involving numbers that are similar to the ones they face in market transactions and the other involving numbers that are order of magnitude larger. As Table 7 shows, 38% of children responded correctly to the first transaction and 27% to the second transaction on their first try, but if we include the children who responded correctly after making mistakes, the corresponding figures are 54% and 50%, respectively. These results provide further evidence that children's performance on the market transactions is not explained by memorization of combinations of prices and quantities. They also confirm that children's performance in written assessments underestimates the types of operations that many of them can perform correctly.

Table 7. Children's performance hypothetical market transactions

	Transactions	
	1	*2*
Child responded correctly	.376	.265
	(.486)	(.443)
Child responded correctly after mistakes	.162	.231
	(.37)	(.423)
Child responded incorrectly	.342	.291
	(.476)	(.456)
Child was helped by someone else	0	0
	(0)	(0)
Child did not try to answer	.12	.214
	(.326)	(.412)
N	117	117

Notes: (1) Table shows means, standard deviation in parenthesis, and number of non-missing observations. (2) Transaction 1: "Suppose that oil is 100 Rupees per kilogram and rice is 80 Rupees per kilogram. How much should I pay for 300 grams of oil and 800 grams of rice?" Transaction 2: "Suppose that meat is 400 Rupees per kilogram and Ilsha is 600 Rupees per kilogram. How much should I pay to buy 400 grams of meat and 600 grams of fish?"

This conclusion is reinforced by examining the heterogeneity among the children. We expected children who sold goods by kilogram to be able to correctly respond to a wider range of arithmetic problems than children who sold goods by unit. We hypothesized that children selling goods by kilogram would find it much harder to memorize combinations of prices and quantities to calculate amounts due (simply because, when dealing with fractions of goods, there are many more possible combinations than when dealing with units of goods).[19] We knew of no prior studies exploring this question.

As Table 8 indicates, we found that children selling goods by kilogram were more likely than those selling goods by units to attempt and to answer correctly the hypothetical market transactions correctly. These differences were only statistically significant in the first transaction (possibly, due to the small number of children asked these questions). These results provide suggestive evidence to support our expectations.

Table 8. Children's performance hypothetical market transactions by type of goods sold

	Transaction 1			*Transaction 2*		
	Unit	*Kilo*	*Diff.*	*Unit*	*Kilo*	*Diff.*
Child responded correctly	.274	.491	-.216**	.209	.327	-.117
	(.449)	(.505)	(.088)	(.410)	(.474)	(.081)
Child responded correctly after mistakes	.129	.2	-.071	.209	.255	-.044
	(.337)	(.404)	(.068)	(.410)	(.44)	(.078)
Child responded incorrectly	.403	.273	.130	.338	.236	.102
	(.494)	(.449)	(.087)	(.477)	(.429)	(.084)
Child was helped by someone else	0	0	0	0	0	0
	(0)	(0)	(0)	(0)	(0)	(0)
Child did not try to answer	.193	.036	.157***	.241	.182	.060
	(.398)	(.189)	(.058)	(.431)	(.389)	(.076)
N	62	55	117	62	55	117

Notes: (1) Columns 1-2 and 4-5 show means, standard deviation in parenthesis, and number of non-missing observations. Columns 3 and 6 show the differences between both groups from a two-tailed t-test and its associated standard error. (2) Transaction 1: "Suppose that oil is 100 Rupees per kilogram and rice is 80 Rupees per kilogram. How much should I pay for 300 grams of oil and 800 grams of rice?" Transaction 2: "Suppose that meat is 400

[19] Note that, like in the case of in- and out-of-school children, this could either be a selection (i.e., children with better arithmetic skills are recruited to sell goods by kilogram) or a treatment effect (i.e., children learn arithmetic skills because they sell goods by kilogram).

Rupees per kilogram and Ilsha is 600 Rupees per kilogram. How much should I pay to buy 400 grams of meat and 600 grams of fish?" (3) * p<.1, ** p<.05 *** p<.01.

Discussion

This paper reported the results of a survey of children working in informal markets in Kolkata, West Bengal. Specifically, it presented three main findings: (a) study participants performed poorly on arithmetic problems as typically presented in school, but excelled at similar problems when framed as market transactions; (b) this discrepancy is not explained by aspects unrelated to children's arithmetic ability, such as their capacity to memorize prices and quantities in market transactions, assistance from others at their shops, reliance on calculation aids, or reading and writing skills; and (c) children could apply their knowledge of arithmetic to correctly respond to market transactions of goods that they did not sell.

These findings raise important questions for research and policy. The main question for research is why children perform so differently on "school" and "market" arithmetic problems. Prior studies offer useful guidance on this question. Reed and Lave (1979) argued that individuals tackle different problems by resorting to different "arithmetic systems" (i.e., a set of number names and strategies for performing addition, subtraction, multiplication, and division).[20] The authors contended that individuals resort to different systems depending on the type of problem that they encounter: they respond to problems dealing with quantities by strategies such as counting fingers or manipulating counters, and they respond to problems requiring the manipulation of symbols by attempting the algorithmic computation strategies taught at school. Reed and Lave concluded that this was why Liberian tailors trained through apprenticeships performed better on job-related arithmetic problems than those trained through formal schooling, but worse on abstract arithmetic problems.

Nunes Carraher et al. (1985) were among the first to note that the differences in arithmetic systems that Reed and Lave had found *between* individuals could also be found *within* individuals. They illustrated this point by focusing on children, who need to solve arithmetic problems everyday outside of school, but are required to use symbolic manipulation at school. They found that Brazilian child street vendors used different computation routines for job-related problems than for abstract arithmetic problems.

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[20] Ginsburg (1982) made a similar distinction between what he called "systems of mathematical knowledge".

An important issue that remains unresolved is why children who know computation routines that allow them to solve job-related problems do not apply these routines to abstract arithmetic problems. Nunes Carraher et al. (1987) examined whether the context in which children encounter arithmetic problems influences the strategies that they use. They found that Brazilian third graders were more likely to adopt oral strategies to solve problems in a simulated shop and more likely to adopt written strategies to solve problems if presented with computation exercises, even if the operations in both conditions were the same. They argued that this occurred because of the different "symbolic systems" across these two contexts (i.e., because most problems in the context of the market are solved orally whereas most problems at school are solved in writing) (for a discussion of symbolic systems, see Vygotsky, 1962; Luria, 1976).

Yet, this explanation is problematic for multiple reasons. First, Nunes Carraher and colleagues seem to take as given that the computation routines that children use (e.g., repeated additions to solve a multiplication problem) are tied to the media that children use to apply them (e.g., oral versus written). But there seems to be nothing that keeps children from applying the same routines across different types of media. Second, and perhaps more importantly, Reed and Lave's theory of arithmetic systems, and Nunes Carraher et al.'s theory of symbolic systems, may explain why children (and adults) are more prone to *initially* resort to certain problem-solving strategies when faced with a given problem. Yet, neither theory explains why, upon failing to solve arithmetic problems with school-prescribed routines, individuals who know other strategies do not resort to them. This remains a key barrier to understanding why simply knowing multiple computation routines is not enough.

The main question for policy is whether—and if so, how—the discrepancy in children's performance on "school" and "market" arithmetic problems should inform classroom instruction in India. Prior studies offer useful guidance on this question as well. Nunes Carraher and Schliemann (1985) were among the first to categorize the problem-solving strategies that schoolchildren used to solve addition and subtraction problems, including: counting, school-prescribed routines, grouping quantities, and using previous results.[21] They also discovered that each strategy was associated with certain types of mistakes (see also Nunes Carraher et al., 1987; Nunes et al., 1993).

[21] Similarly, Nunes Carraher et al. (1987) identified two common strategies: repeated grouping and decomposition.

These studies suggest that a promising approach to help the children in our study would be to encourage them to apply the problem-solving strategies that they use in the market transactions to solve computation exercises. How to do this, however, is not entirely clear.[22] One possibility would be to draw connections between computation exercises and market transactions (e.g., if the problem is "80 – 42 =", we could prompt students to "think of 80 as the amount of money tendered by a customer"). The results from our market transactions (both actual and hypothetical) indicate that a non-trivial share of students arrive at the correct answer after the enumerator help them make this type of connection. Another possibility would be to restate the entire computation exercise as a market transaction and measure whether children can subsequently start doing this on their own (e.g., in the above problem, we could say "something costs 42 Rupees, and I gave you 80 Rupees, how many would you give me back?") These strategies may prove effective with working children, but they seem less appropriate to help schoolchildren who do not work and also struggle with abstract arithmetic.

A more general approach would be to use children's errors when attempting to solve arithmetic problems to diagnose their misconceptions. One possibility would be to anticipate the types of errors that children are likely to make when using different problem-solving strategies and warn them of how to avoid common mistakes. Another possibility would be to use a computer-based tutoring or gaming system that assigns children to arithmetic problems specifically selected to help identify their errors and diagnose their potential misconceptions. This is not a new idea. Almost four decades ago, Brown and Burton (1978) proposed a detailed account of how this could be done in arithmetic. However, recent technological advances have vastly lowered the costs and increased the sophistication of algorithms to identify student errors (see, for example, Muralidharan et al., 2017).

[22] Several scholars have suggested that children ought to be encouraged to use different computation routines to check their results, but have provided little specifics on how to help children identify and apply these routines (Nunes Carraher et al., 1985; Nunes Carraher & Schliemann, 1985; Nunes Carraher et al., 1987; Nunes et al., 1993).

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